

The DEA based stochastic evaluation model for supplier's performance over time

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Motivation

- Suppliers' performance is directly related to performance of the overall supply chain of a firm
- Suppliers are located across the globe, and their performance is affected by environments
 - It has resulted in the increased complexity of supplier evaluation.
- A framework for assessment of performance of overseas suppliers by multiple criteria is essential

1. Uses **multiple criteria** such as Cost, Quality and Lead Time.
2. Assumes **stochastic features** because of uncertain environments.
3. Supplier's performances are **changing over time**.

Multiple criteria: DEA

- DEA allows us to assess a set of homogeneous subjects' efficiencies and consider multiple criteria (Charnes, Cooper & Rhodes, 1978)

[CCR-DEA model]

$$\begin{aligned}
 & \max_{z_0, \lambda} z_0 \\
 & s. t. \\
 & - \sum_{j=1}^n y_{rj} \lambda_j + y_{r0} z_0 \leq 0 \quad \forall r \\
 & \sum_{j=1}^n x_{ij} \lambda_j \leq x_{i0} \quad \forall i \\
 & \lambda_j \geq 0 \quad \forall j
 \end{aligned}$$

Measures **the efficiencies** of DMUs

Consider **multiple inputs and outputs**

Stochastic: Chance constrained DEA

- “Chance constrained DEA” which includes probabilistic features. It assumes a normal distribution (Land, Lovell & Thore, 1993)

[CCDEA based on CCR model]

$$\min_{\theta, \lambda} \theta$$

s. t.

$$Prob(Y^n \lambda \leq y_{n0}) \leq p \quad \forall n \quad \longrightarrow \quad p \text{ is the risk level of violating PPS} \\ \text{(e.g., 0.1, 0.05, or 0.01)}$$

$$x_{m0} \geq X^m \lambda \quad \forall m$$

$$PPS(of CCR) = \{(x_o, y_o) \mid x_o \geq X\lambda, y_o \leq Y\lambda, \lambda \geq 0\}$$

Changing over time: MPI

- Malmquist Productivity Index (MPI) based on DEA can trace changes in the efficiencies of DMUs over time while other DEAs just compute the efficiencies at a single point (Fare et al. 1994)

[MPI based SBM model]

$$\delta^s((x_o, y_o)^t) = \min_{s_i^-, s_r^+, \lambda} \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{s_i^-}{x_{io}^t}}{1 + \frac{1}{s} \sum_{r=1}^s \frac{s_r^+}{y_{ro}^t}}$$

s. t.

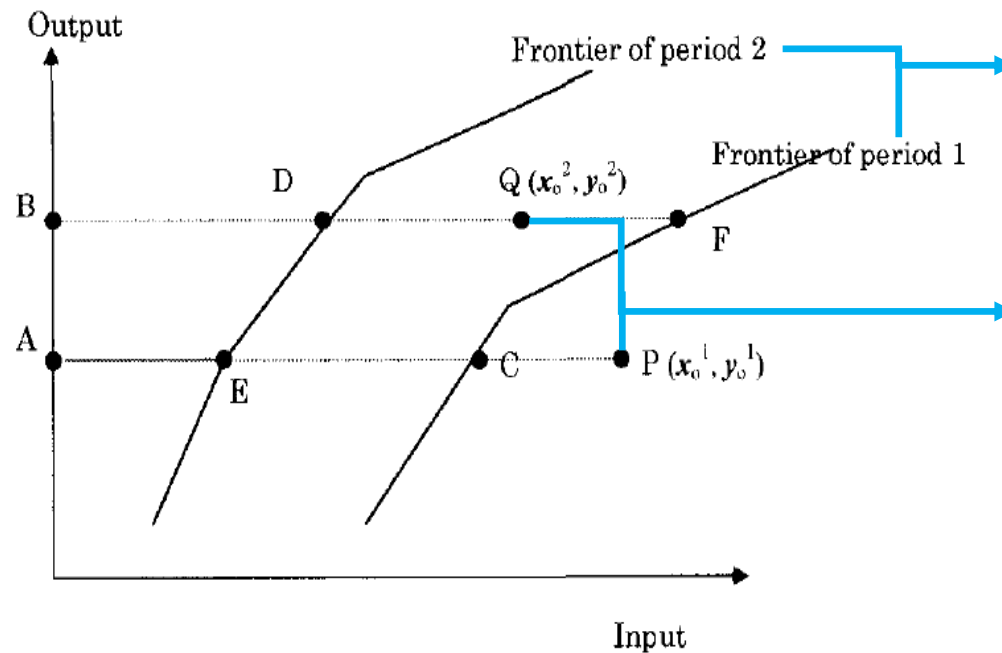
$$x_{io}^t = \sum_{j=1}^n x_{ij}^s \lambda_j + s_i^- \quad \forall i$$

$$y_{ro}^t = \sum_{j=1}^n y_{rj}^s \lambda_j + s_r^+ \quad \forall r$$

$$L \leq e\lambda \leq U$$

$$\lambda_j \geq 0 \forall j, s_i^- \geq 0 \forall i, s_r^+ \geq 0 \forall r$$

(Tone 2004)



Frontier-shift

: it is a **change** in the **efficient frontier** comprised by the **efficient DMUs**.

Catch-up

: it is a **change** in the **efficiency** of **individual DMUs**.

MPI

: **Multiplied Product** of two values in the above.

(Cooper, Seiford and Tone, 2007)

The main contribution of this study

- **MPI** traces changes in the efficiencies of DMUs.
 - **CCDEA** reflects stochastic features in data.
- Proposed model suggests **a standard** how to assess supplier's performances based on methodologies mentioned above.

Main Assumptions

1. **Normal Distribution:** Variability of data follows the normal distribution (Birge & Louveaux, 2011)
2. **Unbiased:** Observed performance is considered as the unbiased estimator of the parameter, that is, the expected value of all inputs and outputs is the same as the parameter (Land, Lovell & Thore, 1993)
$$E(\tilde{x}_{ij}) = x_{ij}, E(\tilde{y}_{rj}) = y_{rj} \quad \forall i, \forall r, \forall j$$
3. **Statistically independent:** All inputs and outputs are statistically independent (Land, Lovell & Thore, 1993)
$$\text{Cov}(x_{ik}, x_{ij}) = 0, \text{Cov}(y_{rk}, y_{rj}) = 0 \quad \forall i, \forall r, \forall k, \forall j$$
4. The model can be infeasible, so “**Super SBM model**” is used to avoid infeasibility (Tone, 2004)

Notations (1/2)

Notation	Description
$i = 1, \dots, m$	i^{th} input factor
$r = 1, \dots, s$	r^{th} output factor
$j = 1, \dots, n$	the subsidiary index (DMUs)
$j = 0$	the DMU under investigation
$t = t_1, t_2$	the time period when values are measured
x_{ij}^t	i^{th} input at period t for DMU j
y_{rj}^t	r^{th} output at period t for DMU j
X^t	input vector at t
Y^t	output vector at t
α	the risk level violating PPS

Notations (2/2)

Notation	Description
s_i^-	the surplus variable (the excesses) of i^{th} input
s_r^+	the shortage variable (the shortfalls) of r^{th} output
σ_{i0}^I	the standard deviation of i^{th} input for DMU j
σ_{r0}^O	the standard deviation of r^{th} output for DMU
ϕ	the cumulative function of standard normal distribution
ϕ^{-1}	the inverse of ϕ
λ	the vector of DMU denoting which DMUs are in the reference set

Base Model - Stochastic SBM Model

- Overview of the Model (Azadi & Saen 2012; Copper, Seiford & Tone 2007)

$$\min_{s_i^-, s_r^+, \lambda} \delta^{t_2}((x_o, y_o)^{t_1}) = \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{s_i^-}{\tilde{x}_{io}^{t_1}}}{1 + \frac{1}{s} \sum_{r=1}^s \frac{s_r^+}{\tilde{y}_{ro}^{t_1}}}$$

Non-radial and non-oriented SBM model which considers inputs and outputs simultaneously

s. t.

$$P\{\tilde{x}_{io}^{t_1} - \bar{X}^{t_2} \lambda \geq s_i^-\} = 1 - \alpha \quad \forall i, j$$

$$P\{\bar{Y}^{t_2} \lambda - \tilde{y}_{ro}^{t_1} \geq s_r^+\} = 1 - \alpha \quad \forall i, j$$

$$L \leq e\lambda \leq U, \quad (L, U) = (0, \infty)$$

$$\lambda \geq 0, \quad s_i^- \geq 0 \quad \forall i, \quad s_r^+ \geq 0 \quad \forall r$$

Stochastic features are reflected in constraints

Constant Return to Scale (RTS) considered

$$\delta^1((x_o, y_o)^1)$$

$$\delta^2((x_o, y_o)^1)$$

$$\delta^1((x_o, y_o)^2)$$

$$\delta^2((x_o, y_o)^2)$$

Four components of MPI are calculated for each DMU

Base Model - Stochastic SBM Model

- Deterministic equivalent of SBM model

$$\min_{\xi_i \varphi_r \lambda} \delta^{t_2}((x_o, y_o)^{t_1}) = \frac{1 - \frac{1}{m} \sum_{i=1}^m \xi_i}{1 + \frac{1}{s} \sum_{r=1}^s \varphi_r}$$

s. t.

$$(1 - \xi_i) \bar{x}_{i0}^{t_1} - \bar{X}^{t_2} \lambda + \phi^{-1}(1 - \alpha) \sigma_i^I(\lambda) = 0 \quad \forall i$$

$$-\bar{Y}^{t_2} \lambda + (1 + \varphi_r) \bar{y}_{r0}^{t_1} - \phi^{-1}(1 - \alpha) \sigma_r^O(\lambda) = 0 \quad \forall r$$

$$L \leq e\lambda \leq U, (L, U) = (0, \infty)$$

$$\lambda \geq 0, \xi_i \geq 0 \quad \forall i, \varphi_r \geq 0 \quad \forall r$$

where,

$$((\sigma_i^I(\lambda))^2 = \sum_{j \neq 0} \lambda_j^2 (\sigma_{ij}^I)^2 + (\lambda_o - 1)^2 (\sigma_{io}^I)^2, \quad \forall i$$

$$((\sigma_r^O(\lambda))^2 = \sum_{j \neq 0} \lambda_j^2 (\sigma_{rj}^O)^2 + (\lambda_o - 1)^2 (\sigma_{ro}^O)^2, \quad \forall r$$

Deterministic equivalents have been obtained by the inverse cumulative function of standard normal distribution

The constraints for inputs and outputs have been transformed into deterministic equivalents

Data variability defined as each factors standard deviation is calculated and inserted in constraints

Alternative Model – Stochastic Super SBM Model

- Deterministic equivalent of Super SBM Model
 - Assumption 4: If SBM is infeasible, **infeasible solutions** can be substituted by feasible **counterparts** which are obtained by **Super SBM** (Tone 2004)

$$\min_{\xi_i \varphi_r \lambda} \delta^{t_2} ((x_o, y_o)^{t_1}) = \frac{1 + \frac{1}{m} \sum_{i=1}^m \xi_i}{1 - \frac{1}{s} \sum_{r=1}^s \varphi_r}$$

s. t.

$$(1 + \xi_i) \bar{x}_{i0}^{t_1} - \bar{X}^{t_2} \lambda + \phi^{-1}(1 - \alpha) \sigma_i^I(\xi, \lambda) = 0 \quad \forall i$$

$$-\bar{Y}^{t_2} \lambda + (1 - \varphi_r) \bar{y}_{r0}^{t_1} + \phi^{-1}(1 - \alpha) \sigma_r^O(\varphi, \lambda) = 0 \quad \forall r$$

$$L \leq e\lambda \leq U, (L, U) = (0, \infty)$$

$$\lambda \geq 0, \xi_i \geq 0 \quad \forall i, \varphi_r \geq 0 \quad \forall r$$

where,

$$((\sigma_i^I(\lambda))^2 = \sum_{j \neq 0} \lambda_j^2 (\sigma_{ij}^I)^2 + (\lambda_o - \xi)^2 (\sigma_{io}^I)^2, \quad \forall i$$

$$((\sigma_r^O(\lambda))^2 = \sum_{j \neq 0} \lambda_j^2 (\sigma_{rj}^O)^2 + (\lambda_o - \varphi)^2 (\sigma_{ro}^O)^2, \quad \forall r$$

In the Super SBM model for MPI,

- ξ_i = the expand rate of input i

- φ_r = the shrinkage rate of output r

(Cooper, Seiford & Tone 2004)

Assessing Standard and MPI formulation

Assessing Standard

- If **MPI > 1**
 \Rightarrow The performance of supplier has been **improved**.
- If **MPI = 1**
 \Rightarrow The performance of supplier has **not** been **changed**.
- If **MPI < 1**
 \Rightarrow The performance of supplier has been **deteriorated**.

MPI Formulation

$$\text{Catch-up} = \frac{\delta^{t_2}((x,y)^{t_2})}{\delta^{t_1}((x,y)^{t_1})}$$

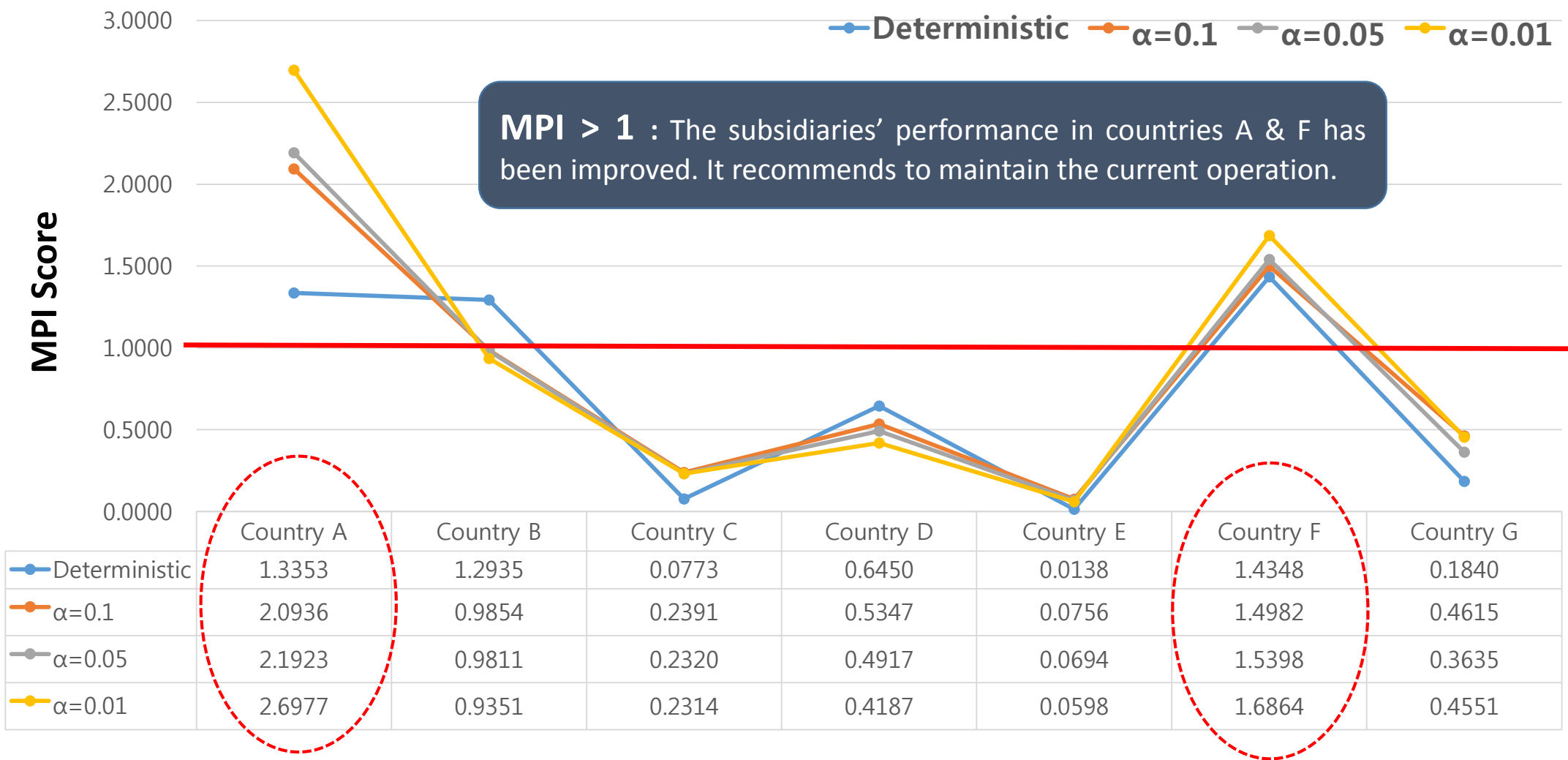
$$\text{Frontier-Shift} = \left[\frac{\delta^{t_1}((x,y)^{t_1})}{\delta^{t_2}((x,y)^{t_1})} \times \frac{\delta^{t_1}((x,y)^{t_2})}{\delta^{t_2}((x,y)^{t_2})} \right]^{0.5}$$

$$\begin{aligned} \text{MPI} &= (\text{Catch-up}) \times (\text{Frontier-shift}) \\ &= \left[\frac{\delta^{t_1}((x,y)^{t_2})}{\delta^{t_1}((x,y)^{t_1})} \times \frac{\delta^{t_2}((x,y)^{t_2})}{\delta^{t_2}((x,y)^{t_1})} \right]^{0.5} \end{aligned}$$

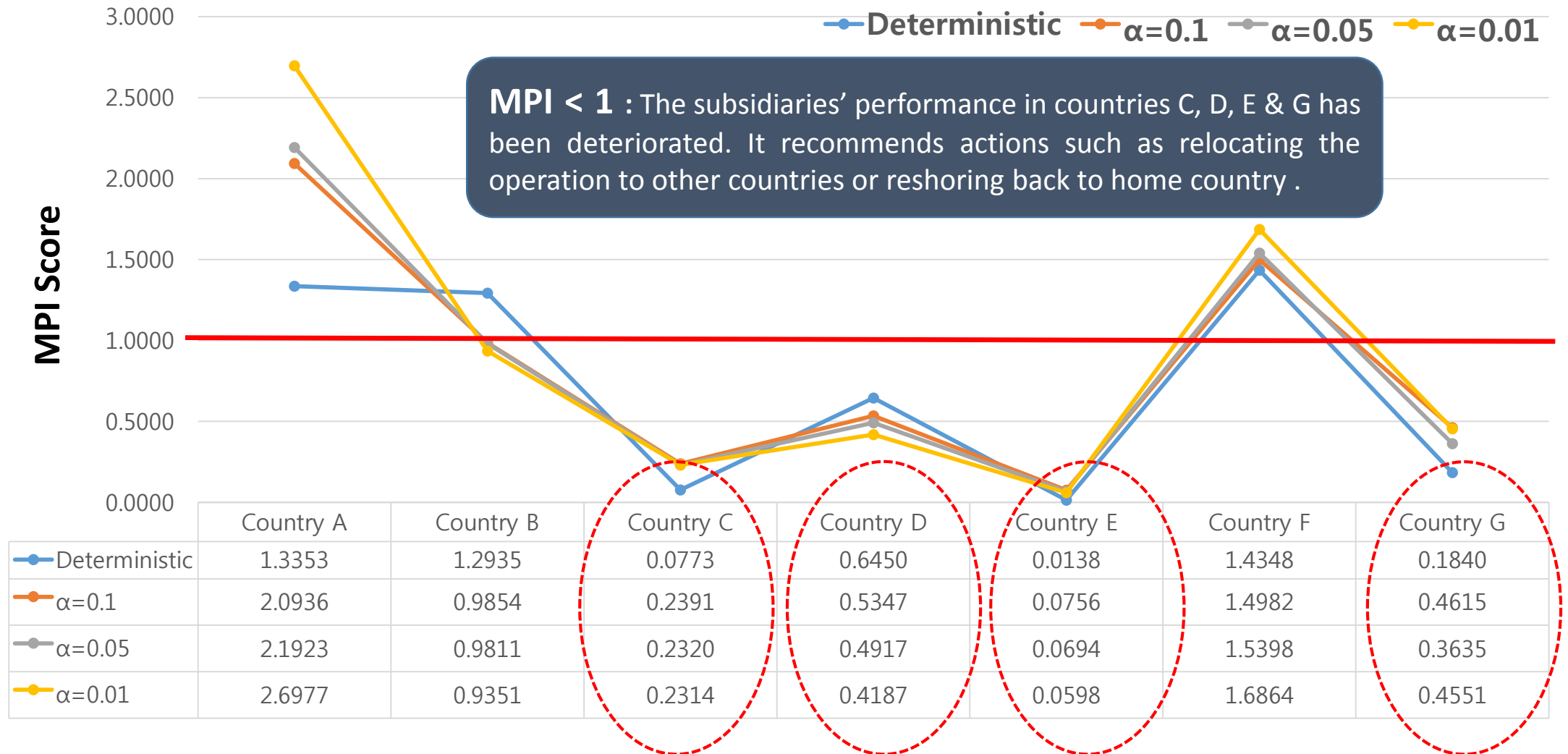
Numerical Example – applying for shoring strategy

- A firm has **seven subsidiaries** in each of the seven overseas countries.
- The performance of the subsidiaries would be evaluated whether it's been improved or not **between two different time periods**.
- **Data** vary with a **normal distribution**.
- The performance of a subsidiary is measured by **two inputs and two outputs**.
 - Input 1: Transportation cost (\$/year)
 - Input 2: Manufacturing cost (\$/year)
 - Output 1: Number of on-time deliveries (/year)
 - Output 2: Number of quality orders (/year)

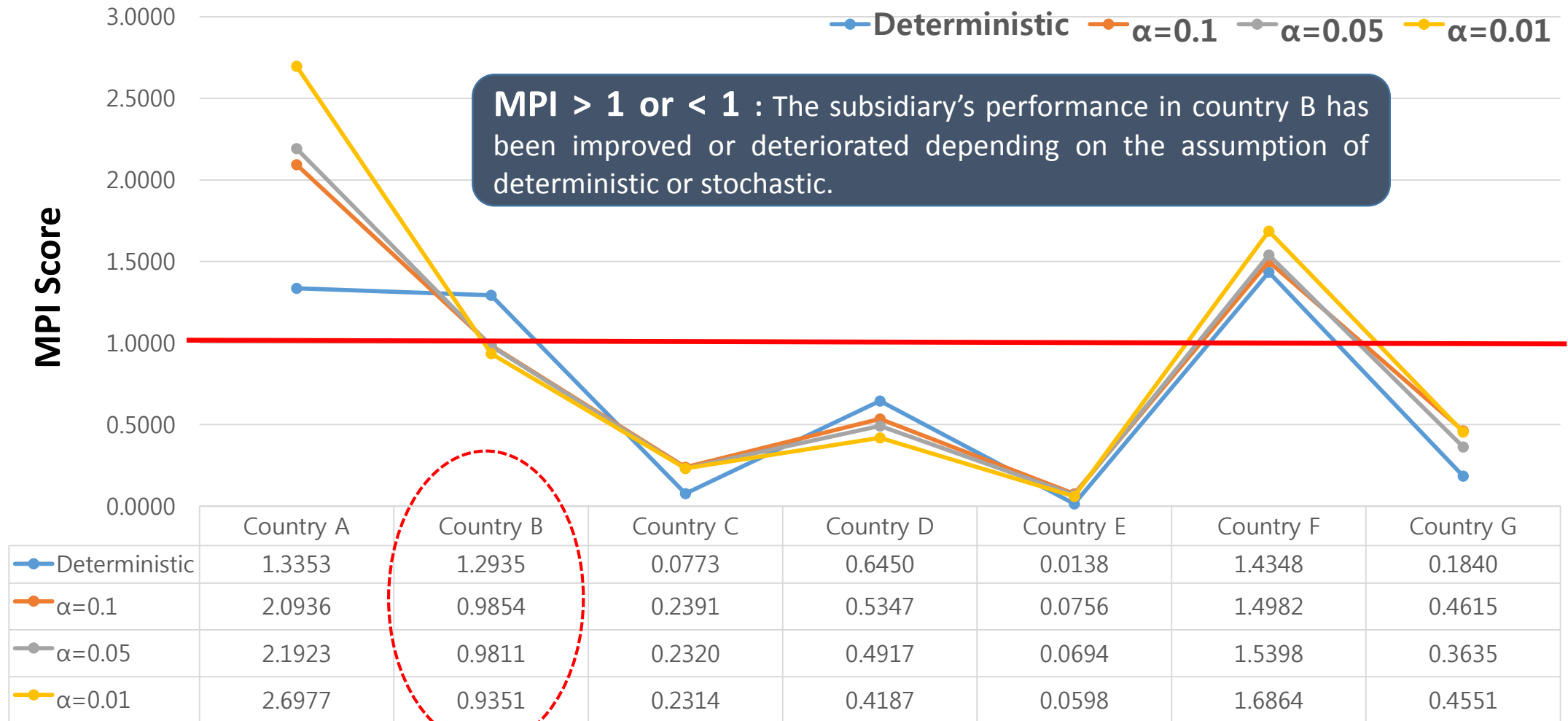
Results: MPI – The changes in efficiency



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Results: MPI – The changes in efficiency



Summary

- The proposed framework enables us to **assess overseas suppliers' performance** of the firm to the current environment compared to the past.
- **Stochastic features** are introduced in MPI based DEA model.
- It is also applicable to the firm's **optimal shoring strategies-offshoring and reshoring decisions**, which is about the firm' factories, operations, etc.

Thank you!