A Stochastic Inventory Model with Carbon Emission Regulations

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Motivations for the models

- There is a growing consensus that carbon emission accelerates global warming.

- The reduction of carbon emission is imperative and governments are under pressure to enact legislation to curb the amount of these emissions.

- Firms are responding to the threat of such legislation or to concerns raised by their own consumers or shareholders and also undertaking initiatives to reduce their carbon footprint.

Motivations for the models (Continued)

- However, these initiatives have mainly focused on energy and logistical efficiency and use of eco-friendly materials from an engineering point of view.
  - Replacement of energy inefficient equipment and facilities, Redesign of products and packaging, Using renewable energy, Establish energy saving process processes

- Or the design of international trade mechanism for carbon emission with an economics perspective.

- Different from the previous studies, I aim to provide optimization models for operational efficiency in a firm by considering various carbon emission regulations.

Main ideas

- Problem setting
  - Random demand
  - Carbon emission regulation: proportional sales tax, symmetric and asymmetric cap-and-trade
  - Single product case
  - Lost-sale model: No shortage cost

- Key contributions
  - Consider various carbon emission regulations
  - Newsvendor analysis: considering underage and overage costs
  - Closed-form solutions and comparative static analysis
  - Numerical examples

Newsvendor approach

- A newsvendor problem is reflective of many real life situation, especially in many industries during a single season.
- Its most unique characteristics are to consider mismatch tradeoff of supply and demand with a combination of underage and overage costs due from random demand.
  - Underage costs: the incremental per-unit cost for not meeting demand
  - Overage costs: the incremental per-unit cost for any items that cannot be sold
- Due to its versatility and simplicity, many variants of the original newsvendor problem have been studied in literature.
- Different from the original newsvendor model, our model adds terms which represent carbon emission regulations. But we still consider underage and overage effects.

Literature review - Deterministic models

- Jung and Jung (2010)
  - Green SCM network design of production, inventory, distribution using multi-period MILP
- Hua *et al.* (2011)
  - Apply EOQ model with carbon emission trading
  - Fixed carbon emission amount and constant unit variable carbon emission for ordering and inventory holding
- Benjaafar *et al.* (2013)
  - Consider various carbon emission regulations (a) hard cap for total carbon emission, (b) proportional sales tax, (c) symmetric cap-and-trade and (d) asymmetric cap-and-trade in a multi-period model
  - It also provide lots of economic insights
- Swami and Shah (2013)
  - A two-echelon SCM model with one supplier and one retailer
  - Each party can make a sustainability effort and it can leads to demand expansion in both ways
  - It also shows that only two-part tariff contract can lead to SCM coordination
- Kim (2017)
  - It uses a multi-objective programming to consider economical and environmental factors and then derive pareto-optimal solutions for green SCM

Literature review - Stochastic models

- It has mainly studied cap-and-trade regulation.
- Min (2015)
  - It extends Swami and Shah (2013) with cap-and-trade regulation in a stochastic model.
  - It studies a Stackelberge game of two-echelon SCM (Leader: carbon emission provider, Follower: manufacturer).
  - It leads to a joint optimization problem of production and sustainability investment.
- Dong *et al.* (2016)
  - It also considers a two-echelon model with cap-and-trade.
  - The sustainability effort by the manufacturer can lead to demand expansion.
- This study
  - It considers various carbon emission regulations in stochastic models.
  - It applies a newsvendor approach in carbon emission regulations.

Model structure

- It extends Benjaafar *et al.* (2013) with a newsvendor approach, which is formulated as stochastic models except hard cap for total carbon emission.
- Model 1: proportional sales tax with lost-sale
- Model 2: symmetric cap-and-trade with lost-sale
- Model 3: asymmetric cap-and-trade with lost-sale

Model structure

- Model parameters
  - p: unit revenue
  - v: salvage value for leftovers
  - c: unit purchasing cost
  - α, β, γ: tax amount per production amount in model 1, 2, 3
  - a: base carbon emission amount with zero production quantity
  - b: additional carbon emission amount per production quantity
  - K: total permissible emission level

- Decision variables
  - x_1, x_2, x_3: production quantity at model 1, 2, 3

- Random variable
  - D: (random) demand from end customer with $F_D(\cdot)$: CDF and $f_D(\cdot)$: PDF.

- In order to prevent trivial solutions, it needs to satisfy that
  - (1) $a \geq 0$, $b \geq 0$ and $K \geq 0$, (2) $\alpha \geq 0$, $\beta \geq 0$ and $\gamma \geq 0$ and
  - (3) $0 \leq v < c < c + \alpha b < p$, $0 \leq v < c < c + \beta b < p$, $0 \leq v < c < c + \gamma b < p$

Model 1: Proportional sales tax with lost-sale

- Profit function

$$\max_{x_1 \geq 0} \mathbb{E}[\Pi_1(x_1, D)] = \mathbb{E}[p \min\{D, x_1\} - cx_1 + v(x_1 - D)^+ - \alpha(a + bx_1)]$$

Then, $x_1^* = F_D^{-1}\left(\frac{p-c-\alpha b}{p-v}\right)$

Model 2: Symmetric cap-and-trade with lost-sale

- Profit function

$$\max_{x_2 \geq 0} \mathbb{E}[\Pi_2(x_2, D)] = \mathbb{E}[p \min\{D, x_2\} - cx_2 + v(x_2 - D)^+ - \beta((a + bx_2) - K)]$$

Then, $x_2^* = F_D^{-1}\left(\frac{p-c-\beta b}{p-v}\right)$

Model 3: Asymmetric cap-and-trade with lost-sale

- Profit function

$$\max_{x_3 \geq 0} \mathbb{E}[\Pi_3(x_3, D)] = \mathbb{E}[p \min\{D, x_3\} - cx_3 + v(x_3 - D)^+ - \gamma((a + bx_3) - K)^+]$$

- Proposition 1. Let us denote that $x_0^* \equiv x_1^*(\alpha = 0) = x_2^*(\beta = 0) = x_3^*(\gamma = 0)$. Then, it should satisfy as follow:
  - If $K \geq a + bx_0^* \Rightarrow x_3^*(\gamma) = x_0^*$
  - Otherwise, $K < a + bx_0^* \Rightarrow x_3^*(\gamma) = x_0^*(\alpha)$, $\gamma = \alpha$

Sensitivity analysis

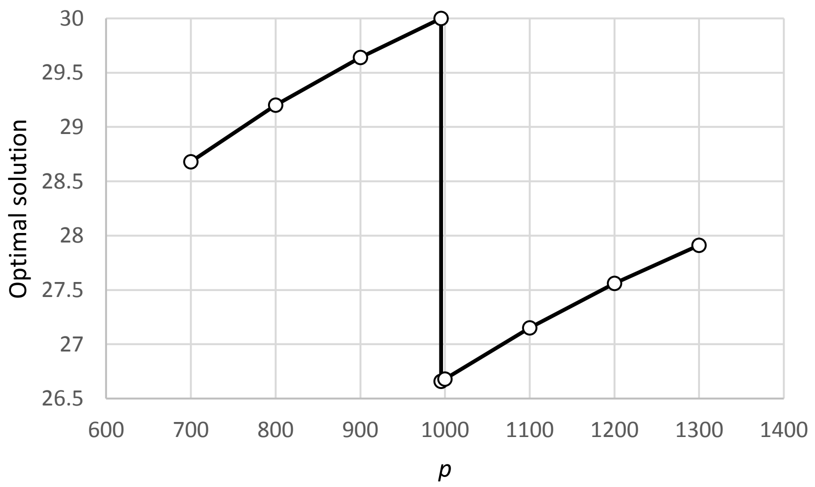
Parameter	Model 1	Model 2	Model 3
p	↑	↑	↑ if $K > a + bx_0^*$ ↓↓ at $K = a + bx_0^*$ ↑ if $K < a + bx_0^*$
v	↑	↑	↑
c	↓	↓	↓ if $K > a + bx_0^*$ ↑↑ at $K = a + bx_0^*$ ↓ if $K < a + bx_0^*$
a	No effect	No effect	No effect
b	↓	↓	No effect if $K \geq a + bx_0^*$ ↓↓ at $K = a + bx_0^*$ ↓ $K < a + bx_0^*$
K	N/A	No effect	No effect
α	↓	N/A	N/A
β	N/A	↓	N/A
γ	N/A	N/A	↓ if $K < a + bx_0^*$ No effect if $K \geq a + bx_0^*$

Numerical example

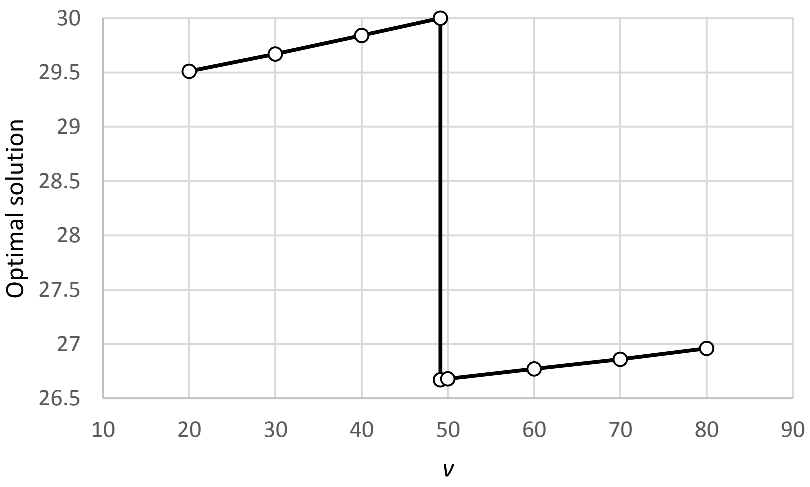
Parameters	Values used
p	1000
v	50
c	200
a	100
b	20
K	700
α	10
β	10
γ	10
D	Normal Distribution with mean 25 and standard deviation 5

Numerical example (Continued)

- Impact of unit revenue

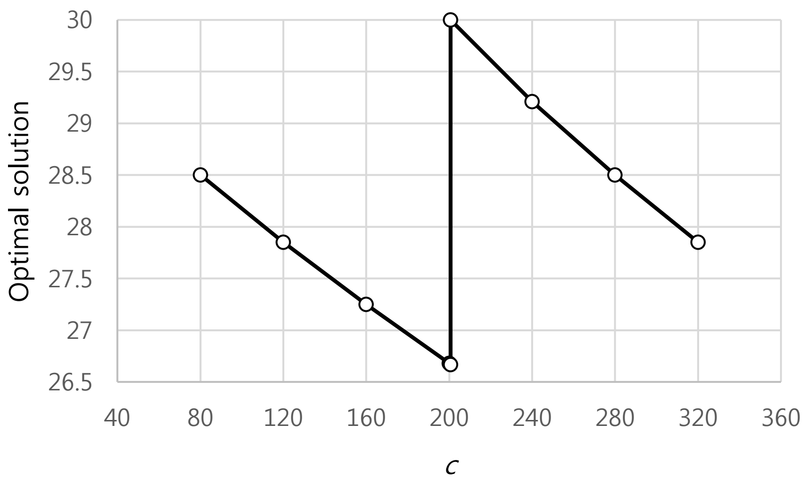


- Impact of salvage value

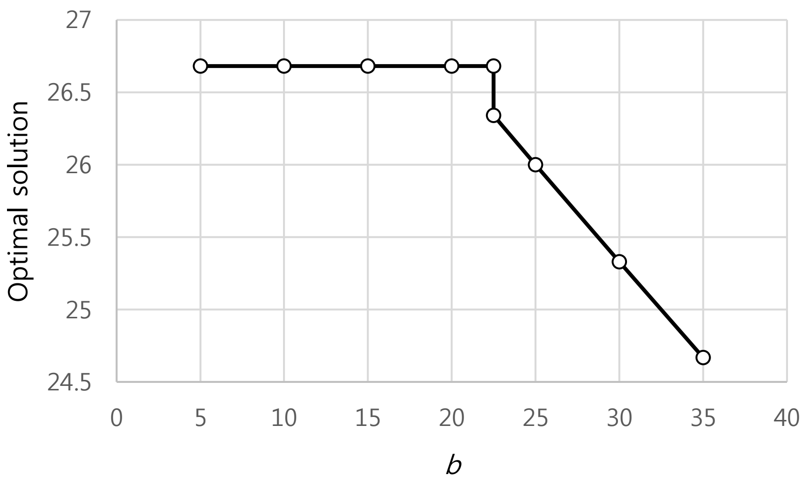


Numerical example (Continued)

- Impact of unit purchasing cost

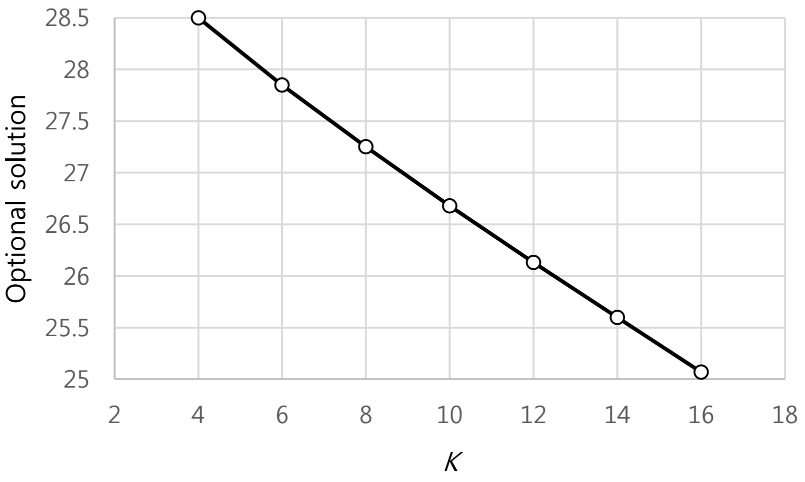


- Impact of additional carbon emission amount per production quantity



Numerical example (Continued)

- Impact of total permissible emission level



Contributions and future studies

- Contributions
  - Provided stylized newsvendor models with various carbon emission regulations
  - Obtained closed-form solutions for the models studied
  - Conducted a comparative static analysis with model parameters
  - Confirmed the analytical results with numerical examples

- Future studies
  - Extends it to SCM model and coordination (Centralized, Decentralized, SCM contracts)
  - Greening efforts may be included (e.g. demand expansion, extra costs)
  - Different risk preferences including risk aversion, loss aversion and so on