퍼지집합론에 기반한 불확실한 수송문제에 대한 절충적 해법

A Compromising Conflict Resolution Approach to the Uncertain Transportation Problem Based on Fuzzy Set Theory

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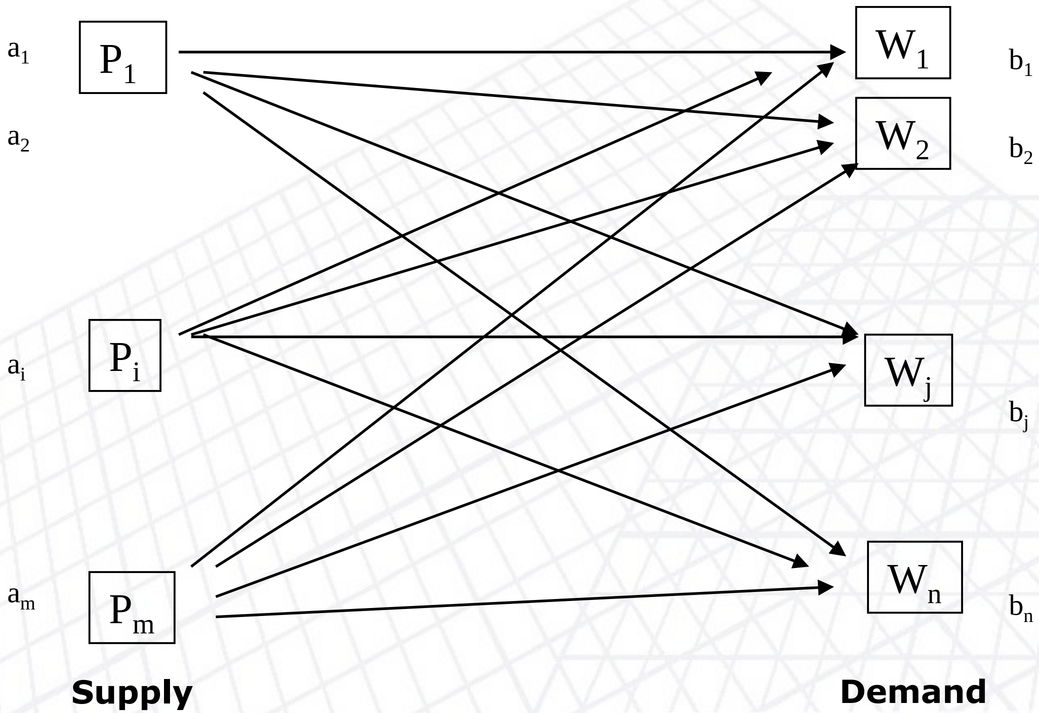
Contents

1. Introduction(1/6)

- Transportation problem is a problem in which products are transported from a set of sources to a set of destinations subject to the supply and demand of the source and the destination respectively, such that the total cost of transportation is minimized.
- Transportation Problem was first developed and proposed by F. L. Hitchcock since 1941



- m- number of sources
- n- number of destinations
- a_i- supply at source i
- b_j – demand at destination j
- c_{ij} – cost of transportation per unit from source i to destination j
- X_{ij} – number of units to be transported from the source i to destination j

1. Introduction(2/6)

- **Transportation Problem Formulation**

The linear programming formulation in terms of the amounts shipped from the sources to the destinations, x_{ij}, can be written as:

$$\min f(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

(total transportation cost)

s.t. $\sum_{i=1}^{m} x_{ij} = S_{i},$ for each source *i* *(supply constraints)*

$\sum_{i=1}^{m} x_{ij} = d_{j},$ for each destination *j (demand constraints)*

$x_{ij} \geq 0$ for all *i* and *j (non-negativity constraints)*

1. Introduction(3/6)

Multi-Objective Transportation Problem Formulation

$$\min\ f_1(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c^1{}_{ij} x_{ij}$$

$$\min\ f_k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c^k{}_{ij} x_{ij}$$

- m- number of sources
- n- number of destinations
- a_i- supply at source i
- b_j – demand at destination j
- c_{ij} – cost of transportation per unit from source i to destination j
- X_{ij} – number of units to be transported from the source i to destination j
- $f_1(x)$ and $f_k(x)$ – objective functions
- k – number of objectives

Subject to

$$\sum_{i=1}^{m} x_{ij} = b_{j},\ j = 1,2,\ldots.,n \qquad \textit{for all } j.$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \text{ and } x_{ij} \geq 0 \quad \textit{for all } i \textit{ and } j \qquad \sum_{j=1}^{n} x_{ij} \leq a_{i},\ i = 1,2,\ldots.,m \qquad \textit{for all } i.$$

Where $c^k{}_{ij}$ *represents the coefficients related to* X_{ij} *variable for objective k*

1. Introduction(4/6)

- **Multi-Objective Interval Transportation Problem(MOITP) Formulation**

$$\text{minimize} \quad Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{\mathrm{L}_{ij}}^k,\ c_{\mathrm{R}_{ij}}^k]x_{ij} \quad \text{where } k = 1, 2, \ldots, K,$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad \sum_{i=1}^{m} x_{ij} = b_j,$$

$$x_{ij} \geqslant 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$$

where $[c_{\mathrm{L}ij}^k, c_{\mathrm{R}ij}^k](k = 1, 2, \ldots, k)$ is an interval representing the uncertain objective for the transportation problem

1. Introduction(5/6)

An interval transportation problem construct the data of supply, demand and objective functions such as cost or other objectives in some intervals

- *Example:*
  - *In standard transportation problem unit transportation cost is constant from each source to each destination*
  - *In reality, it is not constant; it depends on amount of transport quantity and capacity of vehicles, or other factors*
  - *Depending on these factors, the unit transportation cost can vary from one number to another, which can be represented as interval [C1, C2]*

- **Interval** is a set of real numbers with the property that any number that lies between two numbers in the set is also included in the set

$$A = \left[a_{L,} a_R\right] = \{a : a_L \leq a \leq a_R, a \in R\}$$

where a_L and a_R are, respectively, the left and right limits of A

1. Introduction(6/6)

Literature Review

		Solving Method	
		Solution algorithm	Bounds
Non-Intervall	El-Wahed,(2001) “A Multi-objective transportation problem under fuzziness” Fuzzy Sets and Systems 117, 27-33	A fuzzy programming approach for solving MOTP problem Used linear membership function	-
Interval	Keshavrz&Khorram(2011) “A fuzzy bi-criteria transportation problem” Computer&Industrial Engineering 61,947-957	Used bi-level programming approach	Left bound –min, Right bound-max
	Ishibuchi&Tanaka(1990) ”Multiobjective programming in optimization of the interval objective function” Europian Journal of Operation Research 48,219-225	Explanation of order relation which represent the decision maker’s preference between interval profit by the right limit, the left limit, the center and the width of an interval	the right limit, the left limit, the center and the width of an interval
	Patel&Dhodiya,(2017) “N-parties contract based interval Transportation problem and its solution” Industrial Engineering&Management Systems	Used Grey situation decision making theory and Nash bargaining model based method	Left and Right
	Kagade&Bajaj(2010) “Fuzzy method for solving multi-objective assignment problem with interval cost” Journal of Statistics and Mathematics PP-01-09	Used An hyperbolic membership function for the objectives Objective is cost	Center and Right
	Patel&Dhodiya,(2017) “Solving multi-objective interval transportation problem using grey situation decision-making theory based on grey numbers “International Journal of Pure and Applied Mathematics,219-233	Grey situation decision making theory is used to maximize and minimize the objectives	Left and Right
	This study	Used a fuzzy programming approach for solving multi-objective interval TP Used linear membership function to get the optimal compromise solution	Center and Right min Center and Left-max

2. Multi-Objective Interval Transportation Problem(2/1)

$$\min\ f_1(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}[t^k_{Lij}, t^k_{Rij}]x_{ij}$$

$$\max\ f_k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}[\Omega^k_{Lij}, \Omega^k_{Rij}]x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \le a_{i,}\ i = 1,2,\ldots\ldots,m \qquad \textit{for all } i.$$

$$\sum_{i=1}^{m} x_{ij} = b_{j,}\ j = 1,2,\ldots\ldots,n \qquad \textit{for all } j.$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \quad \text{and} \quad x_{ij} \ge 0 \quad \textit{for all } i \textit{ and } j$$

- m- number of sources
- n- number of destinations
- a_i- supply at source i
- b_j _ demand at destination j
- t_{ij} _ delivery time of transportation per unit from source i to destination j
- Ω_{ij} _ profit of transportation per unit from source i to destination j
- X_{ij} _ number of units to be transported from the source i to destination j
- $f_1(x)$ and $f_k(x)$ – objective functions
- k – number of objectives

2. Multi-Objective Interval Transportation Problem(2/2)

- **Order relations for maximization problem**

- This order relation $\leq_{LR}$ represents the decision maker's preference for the alternative with the higher minimum profit and maximum profit. There is many pairs of intervals which cannot be compared by Left and Right bounds. For example, if A = [100, 200] and B=[160,180], then neither A ≤ B nor B ≤ A holds. In this case for profit we prefer B. In order to represent the intuition, the order relation by the center and width of interval defined

$$f_L(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\Omega_{Cij}x_{ij} - \sum_{i=1}^{m}\sum_{j=1}^{n}\Omega_{Wij}x_{ij} \qquad f_C(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\Omega_{ij}x_{ij}$$

- Since the center and the width of interval can be considered as the expected value and the uncertainty of an interval respectively, this order relation represents the decision maker's preference for the alternative with the higher expect value and less uncertainty. See Ishibuchi&Tanaka(1990) "Multiobjective programming in optimization of the interval objective function"

- **Order relations for minimization problem**

$$f_R(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}t_{Cij}x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n}t_{Wij}x_{ij} \qquad f_C(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}t_{ij}x_{ij}$$

- The order relation $<_{CW}$ represents the decision maker's preference for the alternative with the lower expected value and less uncertainty, that is, if $A <_{CW} B$, then *A* is preferred to *B*. See Ishibuchi&Tanaka(1990) "Multiobjective programming in optimization of the interval objective function"

These two objectives can be considered as the maximization or minimization of the worst and the average case respectively

3. Solution Methodology(1/3)

- **The study will use a fuzzy programming approach for solving our model**

- The first step to solve the interval transportation problem is to determine matrixes for Right bound and the Center for minimization objective and Left bound and the Center for maximization objective from interval :
  - Next is to assign, for each objective, two values U^k and L^k as upper and lower bounds, respectively, for the *k*th objective.
    - L^k *is the aspired level of achievement for the objective k*
    - U^k *is the highest acceptable level for achievement for the objective k*
    - $d^k = U^k - L^k$ *is the degradation allowance for the objective k*
  - Once the aspiration levels and degradation allowance for each objective have been specified, we have formed the fuzzy model.

3. Solution Methodology(2/3)

- The main steps of the fuzzy programming technique are as follows:

Step 1: Pick the first objective function and solve it as a single objective transportation problem. Continue this process k times for k different objective functions.

Step 2: Evaluate the *k*th objective function at the K optimal solutions (k = 1, 2,…, K). For each objective function, determine its lower and upper bounds (L^k and U^k) according to the set of optimal solutions.

Step 3: Define the membership function

$$\begin{cases} 1 & \text{if} \quad F^k(x) \le L_k, \\ \dfrac{U_k - F^k(x)}{U_k - L_k} & \text{if} \quad L^k < F^k(x) < U_k, \\ 0 & \text{if} \quad F^k(x) \ge U_k, \end{cases}$$

where $L_k \neq U_k, k = 1,2,\dots,K.$ If $L_k = U_k$, then $\mu_k\left(F^k(x)\right) = 1$ for any value of k

3. Solution Methodology(3/3)

Step 4: Construct the fuzzy programming problem and its equivalent LP problem

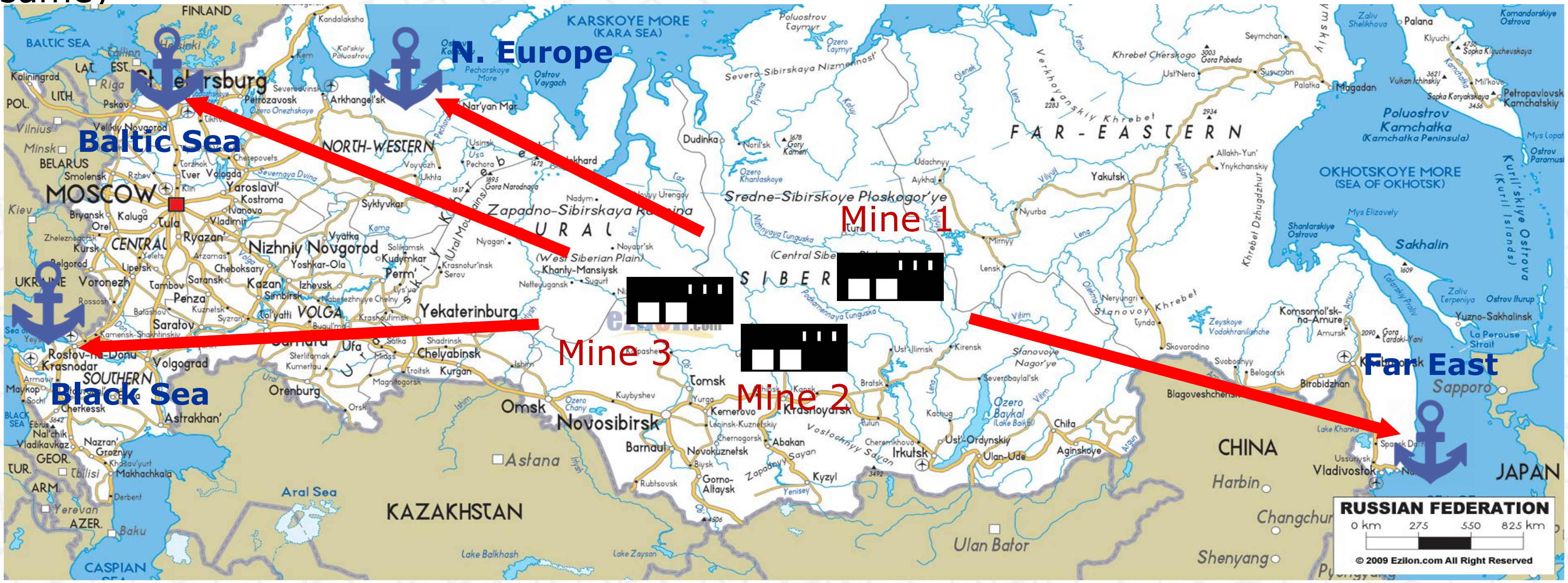
Fuzzy Programming Model	Equivalent LP Problem
Max $\min\limits_{k=1,2,\dots,K} \mu_k(F^k(x))$ Subject to $\sum_{j=1}^{n} x_{ij} = a_i, \quad i=1,2,\dots,m,$ $\sum_{i=1}^{m} x_{ij} = b_j, \quad j=1,2,\dots,n,$ $x_{ij} \geqslant 0, \quad i=1,2,\dots,m,$ $j=1,2,\dots,n.$	Max β *(auxiliary variable)* Subject to $\boxed{\beta \leqslant \mu_k(F^k(x)), \quad k=1,2,\dots,K,}$ $\rightarrow$ $\boxed{\begin{array}{l}\beta(U_k - L_k) \leqslant (U_k - F^k(x)),\\ \beta(U_k - L_k) + F^k(x) \leqslant U_k,\\ \beta(U_k - L_k)/U_k + (1/U_k)F^k(x) \leqslant 1.\end{array}}$ $\sum_{j=1}^{n} x_{ij} = a_i,$ $i=1,2,\dots,m,$ $\sum_{i=1}^{m} x_{ij} = b_j,$ $j=1,2,\dots,n,$ $0 \leqslant \beta \leqslant 1,$ $x_{ij} \geqslant 0 \quad \forall i,j.$

Step 5: Solve LP by using an integer programming technique to get an integer optimal solution and evaluate the K objective functions at this optimal compromise solution.

4. Numerical Example(1/12)

- Data Source from Patel and Dhodiya (2017) and is applied to SUEN company of the Russian coal industry

- Russian coal producer SUEN plans exports of coal produced at the company in September. SUEN has 3 coal mines and produced coal is exported into 4 directions : Far East, Baltic Sea, Black Sea and Northern Europe.
- Based on transportation delivery time and profit, producer shall decide to which direction is better to sell coal and in which quantity (assumption: buying price in all directions is same)



4. Numerical Example(2/12)

Objective 1(Delivery Time) -day

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	[1,2]	[1,3]	[5,9]	[4,8]	8
Mine 2	[1,2]	[7,10]	[2,6]	[3,5]	19
Mine 3	[7,9]	[7,11]	[3,5]	[5,7]	17
Demand (MT)	11	3	14	16	

Objective 2(Profit) –million $

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	[3,5]	[2,6]	[2,4]	[1,5]	8
Mine 2	[4,6]	[7,9]	[7,10]	[9,11]	19
Mine 3	[4,8]	[1,3]	[3,6]	[1,2]	17
Demand (MT)	11	3	14	16	

4. Numerical Example(3/12)

- **Solution**

Step 1: Pick every objective functions and solve as a single-objective transportation problem (right bound for time and left bound for profit)

Objective 1 (Delivery Time) Right bound-min

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	2	3	9	8	8
Mine 2	2	10	6	5	19
Mine 3	9	11	5	7	17
Demand (MT)	11	3	14	16	

Objective 2 (Profit) Left bound-max

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	3	2	2	1	8
Mine 2	4	7	7	9	19
Mine 3	4	1	3	1	17
Demand (MT)	11	3	14	16	

4. Numerical Example(4/12)

Step 1: Pick every objective functions and solve as a single-objective transportation problem (for center)

Objective 1(Time)

	Far East
Mine 1	[1,2]
Mine 2	[1,2]

(1+2)/2

Objective 1 (Delivery Time) Center

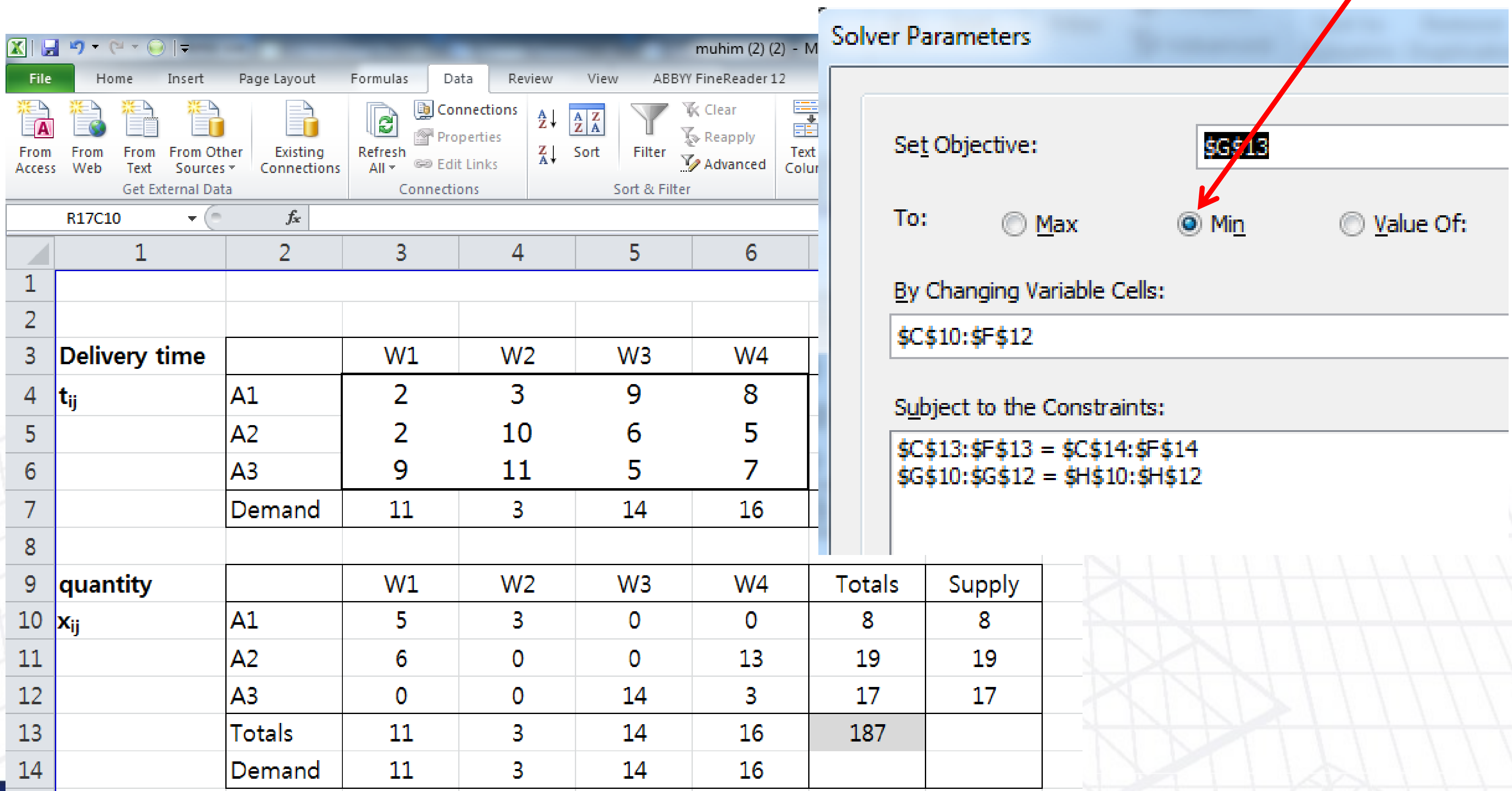
	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	1,5	2	7	6	8
Mine 2	1,5	8,5	4	4	19
Mine 3	8	9	4	6	17
Demand (MT)	11	3	14	16	

Objective 2 (Profit) Center

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	4	4	3	3	8
Mine 2	5	8	8,5	10	19
Mine 3	6	2	4,5	1,5	17
Demand (MT)	11	3	14	16	

4. Numerical Example(5/12)

- Pick objectives delivery time function with Center and Right bounds and solve in the Excel by minimizing and by maximizing profit's Center and Left bounds.



Delivery time		W1	W2	W3	W4
t_{ij}	A1	2	3	9	8
	A2	2	10	6	5
	A3	9	11	5	7
	Demand	11	3	14	16

quantity		W1	W2	W3	W4	Totals	Supply
x_{ij}	A1	5	3	0	0	8	8
	A2	6	0	0	13	19	19
	A3	0	0	14	3	17	17
	Totals	11	3	14	16	187	
	Demand	11	3	14	16		

4. Numerical Example(6/12)

Step 2: For each objective function, determine its lower and upper bounds (L^k and U^k) according to the set of optimal solutions

$F^1(X^{tR})$ = (187, 187, 226, 281) ,

$F^2(X^{tC})$ = (149, 149, 182, 229),

$F^3(X^{pL})$ = (207, 207, 243, 243) ,

$F^4(X^{pC})$ = (260, 260, 303, 306)

i.e. $149 \le F^1 \le 260$, $149 \le F^2 \le 260$, $182 \le F^3 \le 303$, $229 \le F^4 \le 306$

Step 3: Define the membership function

$$\mu_1(F^1(x)) = \begin{cases} 1 & \text{if} \quad F^1(x) \le 149, \\ \dfrac{260 - F^1(x)}{111} & \text{if} \quad 149 < F^1(x) < 260, \\ 0 & \text{if} \quad F^1(x) \ge 260, \end{cases}$$

$$\mu_2(F^2(x)) = \begin{cases} 1 & \text{if} \quad F^2(x) \le 149, \\ \dfrac{260 - F^2(x)}{111} & \text{if} \quad 149 < F^2(x) < 260, \\ 0 & \text{if} \quad F^2(x) \ge 260, \end{cases}$$

$$\mu_3(F^3(x)) = \begin{cases} 1 & \text{if} \quad F^3(x) \le 182, \\ \dfrac{303 - F^1(x)}{121} & \text{if} \quad 182 < F^3(x) < 303, \\ 0 & \text{if} \quad F^3(x) \ge 303, \end{cases}$$

$$\mu_4(F^4(x)) = \begin{cases} 1 & \text{if} \quad F^4(x) \le 229, \\ \dfrac{306 - F^4(x)}{77} & \text{if} \quad 229 < F^4(x) < 260, \\ 0 & \text{if} \quad F^4(x) \ge 306, \end{cases}$$

$\mu_1(F^1(x))$ = (260- $F^1(x)$)/(260– 149)

$\mu_2(F^2(x))$ = (260- $F^2(x)$)/(260 – 149)

$\mu_3(F^3(x))$ = (303- $F^3(x)$)/(303 – 182)

$\mu_4(F^4(x))$ = (306- $F^4(x)$)/(306– 229)

4. Numerical Example(7/12)

Step 4: Construct the fuzzy programming problem and its equivalent LP problem

$$\text{Max} \quad \min_{k=1,2,\ldots,K} \mu_k(F^k(x))$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\ldots,m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\ldots,n,$$

$$x_{ij} \geqslant 0, \quad i = 1,2,\ldots,m,$$

$$j = 1,2,\ldots,n.$$

Membership functions in our case

$\mu_2(F^2(x)) = (260 - F^2(x))/(260 - 149)$

$\mu_1(F^1(x)) = (260 - F^1(x))/(260 - 149)$

$\mu_3(F^3(x)) = (303 - F^3(x))/(303 - 182)$

$\mu_4(F^4(x)) = (306 - F^4(x))/(306 - 229)$

4. Numerical Example(8/12)

By introducing an auxiliary variable β, fuzzy programming problem can be transformed into the following equivalent linear programming (LP) problem

$$
\begin{aligned}
\text{Max} \quad & \beta \\
\text{Subject to} \quad & \left.\begin{aligned} & x_{11}+x_{12}+x_{13}+x_{14}=8, \\ & x_{21}+x_{22}+x_{23}+x_{24}=19, \\ & x_{31}+x_{32}+x_{33}+x_{34}=17, \end{aligned}\right\} \textit{(supply constraints)} \\
& \left.\begin{aligned} & x_{11}+x_{21}+x_{31}=11, \\ & x_{12}+x_{22}+x_{32}=3, \\ & x_{13}+x_{23}+x_{33}=14, \\ & x_{14}+x_{24}+x_{34}=16, \end{aligned}\right\} \textit{(demand constraints)}
\end{aligned}
$$

Objective 1 (Delivery Time) Right bound-min

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	2	3	9	8	8
Mine 2	2	10	6	5	19
Mine 3	9	11	5	7	17
Demand (MT)	11	3	14	16	

Objective 2 (Profit) Left bound-max

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	3	2	2	1	8
Mine 2	4	7	7	9	19
Mine 3	4	1	3	1	17
Demand (MT)	11	3	14	16	

Objective 1 (Delivery Time) Center

(1+2)/2

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	1,5	2	7	6	8
Mine 2	1,5	8,5	4	4	19
Mine 3	8	9	4	6	17
Demand (MT)	11	3	14	16	

Objective 2 (Profit) Center

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	4	4	3	3	8
Mine 2	5	8	8,5	10	19
Mine 3	6	2	4,5	1,5	17
Demand (MT)	11	3	14	16	

$$2x_{11}+3x_{12}+9x_{13}+8x_{14}+2x_{21}+10x_{22}+6x_{23}+5x_{24}+9x_{31}+11x_{32}+5x_{33}+7x_{34}+111\beta \leq 260$$

$$1.5x_{11}+2x_{12}+7x_{13}+6x_{14}+1.5x_{21}+8.5x_{22}+4x_{23}+4x_{24}+8x_{31}+9x_{32}+4x_{33}+6x_{34}+111\beta \leq 260$$

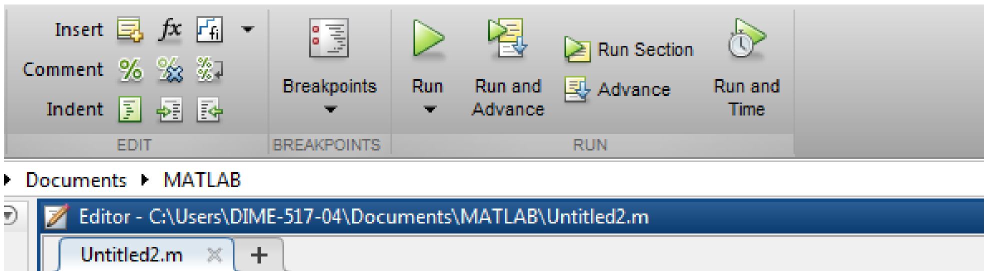
$$3x_{11}+2x_{12}+2x_{13}+x_{14}+4x_{21}+7x_{22}+7x_{23}+9x_{24}+4x_{31}+x_{32}+3x_{33}+x_{34}+121\beta \leq 303$$

$$4x_{11}+4x_{12}+3x_{13}+3x_{14}+5x_{21}+8x_{22}+8.5x_{23}+10x_{24}+6x_{31}+2x_{32}+4.5x_{33}+1.5x_{34}+77\beta \leq 306$$

$$x_{ij} \geqslant 0 \text{ and integer } \forall i,j.$$

4. Numerical Example(9/12)

Step 5: Solve LP by using an integer programming technique to get an integer optimal solution and evaluate the objective Delivery Time and Profit functions at this optimal compromise solution.




```
clear;clc;
f=[0 0 0 0 0 0 0 0 0 0 0 0 -1]';
Aeq=[1 1 1 1 0 0 0 0 0 0 0 0 0;
     0 0 0 0 1 1 1 1 0 0 0 0 0;
     0 0 0 0 0 0 0 0 1 1 1 1 0;
     1 0 0 0 1 0 0 0 1 0 0 0 0;
     0 1 0 0 0 1 0 0 0 1 0 0 0;
     0 0 1 0 0 0 1 0 0 0 1 0 0;
     0 0 0 1 0 0 0 1 0 0 0 1 0];
beq=[8 19 17 11 3 14 16];
A=[2 3 9 8 2 10 6 5 9 11 5 7 111;
   1.5 2 7 6 1.5 8.5 4 4 8 9 4 6 111;
3 2 2 1 4 7 7 9 4 1 3 1 121;
4 4 3 3 5 8 8.5 10 6 2 4.5 1.5 77];
b=[260 260 303 306];
intcon=[1 2 3 4 5 6 7 8 9 10 11 12];
lb=[0 0 0 0 0 0 0 0 0 0 0 0 0]';
ub=[inf inf inf inf inf inf inf inf inf inf inf inf 1]';
x=intlinprog(f,intcon,A,b,Aeq,beq,lb,ub)
```

4. Numerical Example(10/12)

Command Window

```
LP:                 Optimal objective value is -0.643813.

Cut Generation:     Applied 1 Gomory cut.
                    Lower bound is -0.630631.
                    Relative gap is 0.00%.


Optimal solution found.

Intlinprog stopped at the root node because the objective value is within a gap tolerance of the optimal value,
options.TolGapAbs = 0 (the default value). The intcon variables are integer within tolerance,
options.TolInteger = 1e-05 (the default value).


x =

    4.0000
    3.0000
         0
    1.0000
    7.0000
         0
         0
   12.0000
         0
         0
   14.0000
    3.0000
    0.6306
```

The problem is solved using Matlab package, yeilding the following optimal compromise solution X for MOTP problem:

	Far East	Baltic Sea	Black Sea	N. Europe	Supply
Mine 1	4	3	0	1	8
Mine 2	7	0	0	12	19
Mine 3	0	0	14	3	17
Demand	11	3	14	16	

4. Numerical Example(11/12)

The result is obtained as X =[4, 3, 0, 1, 7, 0, 0, 12, 0, 0, 14, 3)

	Far East	Baltic Sea	Black Sea	N. Europe	Supply
Mine 1	4	3	0	1	8
Mine 2	7	0	0	12	19
Mine 3	0	0	14	3	17
Demand	11	3	14	16	

The objective (Delivery time and Profit) function values for each objective are (Delivery time) = [151, 190] and (Profit)= [200,254] , the overall satisfaction β=0.6306

	W1	W2	W3	W4	Supply
A1	1,5	2	7	6	8
A2	1,5	8,5	4	4	19
A3	8	9	4	6	17
Demand	11	3	14	16	

	W1	W2	W3	W4	Totals	Supply
A1	4	3	0	1	8	8
A2	7	0	0	12	19	19
A3	0	0	14	3	17	17
Totals	11	3	14	16	151	
Demand	11	3	14	16		

	W1	W2	W3	W4	Supply
A1	2	3	9	8	8
A2	2	10	6	5	19
A3	9	11	5	7	17
Demand	11	3	14	16	

	W1	W2	W3	W4	Totals	Supply
A1	4	3	0	1	8	8
A2	7	0	0	12	19	19
A3	0	0	14	3	17	17
Totals	11	3	14	16	190	
Demand	11	3	14	16		

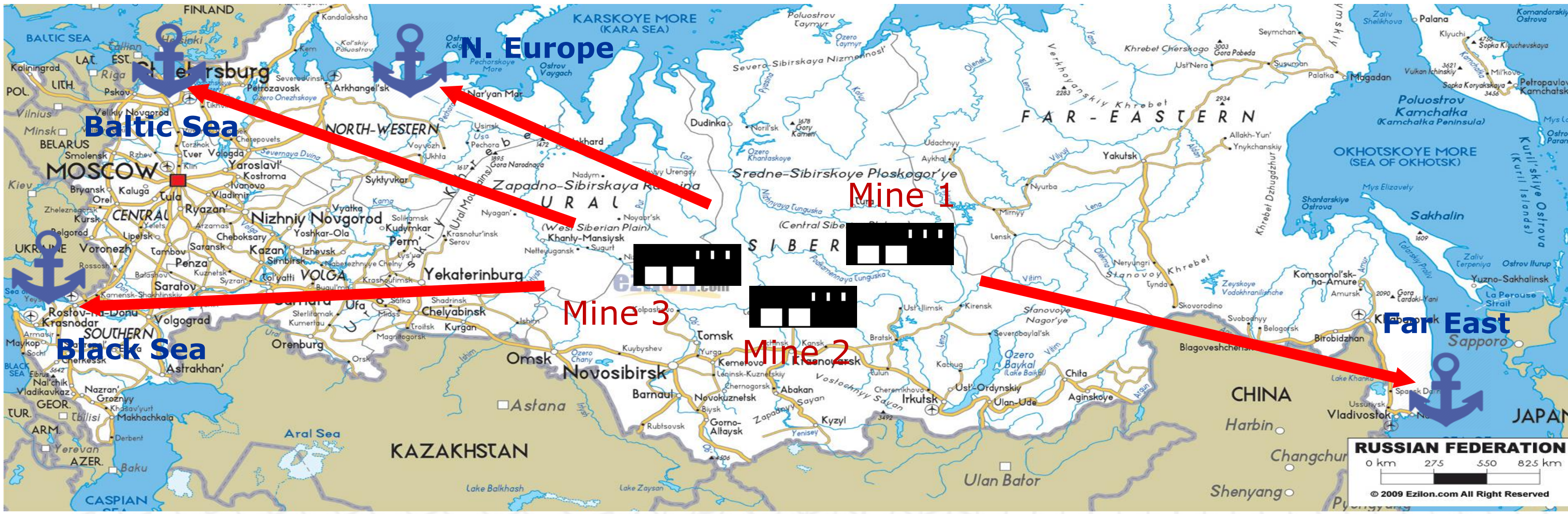
	W1	W2	W3	W4	Supply
A1	3	2	2	1	8
A2	4	7	7	9	19
A3	4	1	3	1	17
Demand	11	3	14	16	

	W1	W2	W3	W4	Totals	Su
A1	4	3	0	1	8	
A2	7	0	0	12	19	
A3	0	0	14	3	17	
Totals	11	3	14	16	200	
Demand	11	3	14	16		

	W1	W2	W3	W4	Supply
A1	4	4	3	3	8
A2	5	8	8,5	10	19
A3	6	2	4,5	1,5	17
Demand	11	3	14	16	

	W1	W2	W3	W4	Totals	Supply
A1	4	3	0	1	8	8
A2	7	0	0	12	19	19
A3	0	0	14	3	17	17
Totals	11	3	14	16	254	
Demand	11	3	14	16		

4. Numerical Example(12/12)



Thus, optimal solution for this case can be achieved by:

- Coal from Mine1 shall be exported to Far East only (4 million tons) and Baltic sea (3 million tons), N. Europe (1 million tons)
- From Mine 2, coal shall be exported to Far East (7 million tons) and N. Europe (12 million tons)
- Mine 3 shall export to Black Sea (14 million tons) , N. Europe (3 million tons)

5. Conclusion

- This study is on the transportation problem with multiple-objectives with intervals. It seeks to solve the transportation problem with two objectives: the minimization of delivery time and maximization of profits of transportation.
- The method to be implemented for solving the problem is a fuzzy programming technique. This approach allows to reach a compromise solution to transportation problem with the given two objectives of delivery time minimization and profit maximization.
- As a future research, comparisons between previous studies and this study and development of another approach will be performed. Also, a case study for a Russian coal producing company, Suen will be planned.

THANK YOU!