

두 단계 재고모형 시스템에서의 최적 세일 시점 결정 문제

Youngchul Shin^a, Chang Seong Ko^b, Ilkyeong Moon^{a, c*}

Department of Industrial Engineering, Seoul National University

Department of Industrial and Management Engineering, Kyungsung University

Institute for Industrial Systems Innovation, Seoul National University

Korean Society of Supply Chain Management

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Outline

I. Introduction

II. Problem description

III. Decentralized system

IV. Centralized system

V. Conclusions

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■ Perishable items

- Finite or limited shelf life
 - ✓ precipitous drop in value
 - ✓ cannot be sold after a certain time
 - ex) newspaper, food, . . .
 - ex) airplane seat, hotel room, fashion good, . . .

■ Newsvendor model

- Conventional approach used to cope with perishable items
 - ✓ single period
 - ✓ uncertain demand
- Provide an optimal order quantity

- Markdown sale

- By reducing the price, more demands can be generated.
ex) supermarket, fashion industry, . . .

- Price-setting newsvendor model

- Using inverse relationship of the price and demand

⇒ Determine the optimal price and order quantity simultaneously.

- Discount rate
 - From the price-setting newsvendor model, an optimal discount rate for markdown sale can be obtained.
- ✓ In practice, the discount rate of a markdown sale, such as 30%, 40%, or 50%, is often predetermined.



■ Research question

What is the *optimal time* to reduce the price?

■ An appropriate start time of price reduction

- Remaining inventory ↓
- Revenue ↑
- Impact on the retailer's order quantity
- Impact on the profits of the manufacturer and supply chain system

⇒ Determine the *optimal start time* of the markdown and *order quantity*.

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■ Assumptions

1. A single retailer places an order to a single manufacturer.
2. The retailer determines the start time of markdown sale t_m and order quantity q .
3. The period is divided into two parts as $[0, t_m]$ and $[t_m, T]$.
4. Until t_m , items are sold at a selling (regular) price, which is subsequently decreased with a discount rate $\alpha \in (0, 1)$ after t_m .

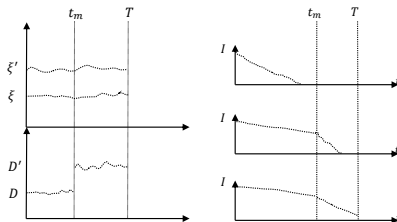
■ Uncertain demands

$\xi = y(p) + \epsilon$, where $y(p) = a - bp$	Demand in $[0, T]$ when the item is sold at the regular price
$\xi' = y(p, \alpha) + \epsilon$, where $y(p, \alpha) = a - (1 - \alpha)bp$	Demand in $[0, T]$ when the item is sold at the sale price
$D = \frac{t_m}{T}\xi = \frac{t_m}{T}y(p) + \frac{t_m}{T}\epsilon$	Demand in $[0, t_m]$ when the item is sold at the regular price
$D' = \left(\frac{T-t_m}{T}\right)\xi' = \frac{T-t_m}{T}y(p, \alpha) + \frac{T-t_m}{T}\epsilon$	Demand in $[t_m, T]$ when the item is sold at the sale price

$y(p)$ and $y(p, \alpha)$: general price dependent functions

$\epsilon \sim N(0, \sigma^2)$: price independent random variable following a normal distribution

■ Uncertain demands and three types of inventory levels



■ Limit of the start time of the markdown sale

$$\lim_{t_m \rightarrow T-} D = \lim_{t_m \rightarrow T-} \frac{t_m}{T} \xi = \lim_{t_m \rightarrow T-} \left(\frac{t_m}{T} y(p) + \frac{t_m}{T} \epsilon \right) = y(p) + \epsilon = \xi$$

$$\lim_{t_m \rightarrow 0+} D' = \lim_{t_m \rightarrow 0+} \left(\frac{T - t_m}{T} \right) \xi' = \lim_{t_m \rightarrow 0+} \left(\frac{T - t_m}{T} y(p, \alpha) + \frac{T - t_m}{T} \epsilon \right) = y(p, \alpha) + \epsilon = \xi'$$

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■ Retailer

- Determines the start time of the markdown sale and the order quantity

q^* : optimal order quantity when t_m is given

t_m^* : optimal start time of the markdown sale when q is given

(t_m^{**}, q^{**}) : optimal combination of start time of the markdown sale and the order quantity

■ Manufacturer

- Depends on the order quantity determined by the retailer

■ Profit function of a retailer

$$\begin{aligned}\Pi_r(q, t_m) &= R_1(q, t_m) + R_2(q, t_m) - C \\ &= p \cdot \mathbb{E}[\min(q, D)] + (1 - \alpha) \cdot p \cdot \mathbb{E}[\min((q - D)^+, D')] - c_r q - wq\end{aligned}$$

$$R_1(q, t_m) = p \cdot \mathbb{E}[\min(q, D)]$$

$$R_2(q, t_m) = (1 - \alpha) \cdot p \cdot \mathbb{E}[\min((q - D)^+, D')]$$

$$C = c_r q + wq$$

■ Profit function of a manufacturer

$$\Pi_m(q) = wq - c_m q$$

Proposition 3.1

The expected profit function of the retailer Π_r is *strictly concave respect to q* where $q \geq 0$ and given $t_m \in [0, T]$. Therefore, there exists a unique q^* maximizing the expected profit function Π_r when t_m is given.

Proposition 3.2

When an optimal order quantity $q^*(t_m)$ is given, *critical ratio (fractile)* $\frac{p - (c_r + w)}{p}$ can be expressed as a *convex combination* of $F(\frac{T}{t_m} q^* - (a - bp))$ and $F(q^* - (a - bp) - \frac{T - t_m}{T} \alpha bp)$.

$$\alpha F\left(\frac{T}{t_m} q^* - (a - bp)\right) + (1 - \alpha) F\left(q^* - (a - bp) - \frac{T - t_m}{T} \alpha bp\right) = \frac{p - (c_r + w)}{p}$$

Corollary 3.3

An optimal order quantity of the retailer has the *lower and upper bounds* shown in the following inequality.

$$\frac{t_m}{T} F^{-1} \left(\frac{p - (c_r + w)}{p} \right) + \frac{t_m}{T} (a - bp) \leq q^* \leq F^{-1} \left(\frac{p - (c_r + w)}{p} \right) + a - bp + \frac{T - t_m}{T} \alpha bp$$

\Rightarrow Lower and upper bounds of q^* are obtained.

\Rightarrow These bounds can be utilized to obtain q^* through the bi-section method.

Proposition 3.4

The expected profit function of the retailer Π_r is *strictly concave with respect to t_m* where $t_m \in [0, T]$ and given $q \geq 0$. Thus, there exists a unique t_m^* maximizing the expected profit function Π_r when q is given.

Proposition 3.5

The expected profit function of the retailer Π_r is strictly concave with respect to q and t_m where $t_m \in [0, T]$ and $q \geq 0$. There exists a unique combination (t_m^{**}, q^{**}) maximizing the expected profit function of the retailer Π_r .

■ Solution procedure for an optimal combination (t_m^{**}, q^{**})

Algorithm 1 Bi-section method algorithm

```

while  $|v(q, t_m)| \leq TOL$  do
     $t_m \leftarrow (LT + UT)/2$ 
    while  $|u(q, t_m)| \leq TOL$  do
         $q \leftarrow (LB + UB)/2$ 
        if  $u(LB, t_m) \cdot u(q, t_m) < 0$  then
             $UB \leftarrow q$ 
        end
        else
             $LB \leftarrow q$ 
        end
    end
     $q^* \leftarrow q$ 
    return  $q^*, \Pi_r(q^*, t_m)$ 
     $t_m^* \leftarrow \arg \max \Pi_r(q^*, t_m)$ 
    if  $v(q^*, LT) \cdot v(q^*, t_m^*) < 0$  then
         $UT \leftarrow t_m^*$ 
    end
    else
         $LT \leftarrow t_m^*$ 
    end
end
end

```

$$u(g, t_m) = p - \alpha p F\left(\frac{T}{t_m}q - (a - bp)\right) - (1 - \alpha)p F\left(q - (a - bp) - \frac{T - t_m}{T}\alpha bp\right) - (c_r + w)$$

$$v(g, t_m) = (a - bp)F\left(\frac{T}{t_m}q - (a - bp)\right) - (1 - \alpha)bp F\left(q - (a - bp) - \frac{T - t_m}{T}\alpha bp\right) + \int_{-\infty}^{\frac{T}{t_m}q - (a - bp)} xf(x)dx$$

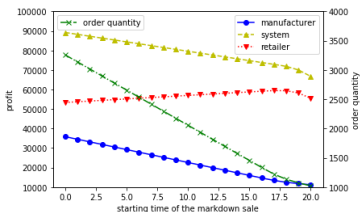
- Profit function of a manufacturer

$$\Pi_m(q) = wq - c_m q$$

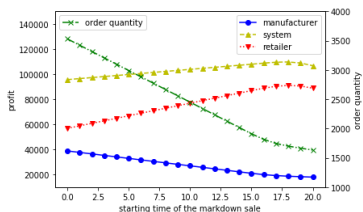
⇒ Proportional to the order quantity determined by the retailer

⇒ Prefer that the retailer starts the markdown sale at $t_m = 0$.

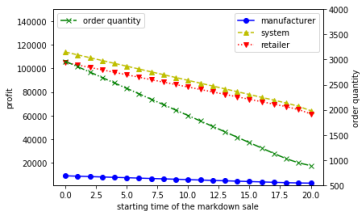
■ Numerical experiments of the decentralized system



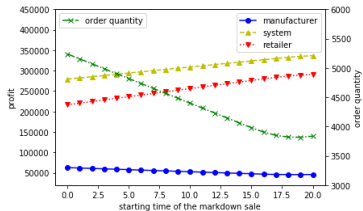
(a) Case 1



(b) Case 2



(c) Case 3



(d) Case 4

Observation 1

Start time of the markdown sale $t_m \uparrow \Rightarrow$ optimal order quantity q^ of the retailer \downarrow*

Observation 2

The retailer will start the markdown sale $t_m = 0$ in Case 3, $t_m = T$ in Case 4, and $t_m = 17.44$ and $t_m = 18.12$, in Cases 1 and 2, respectively.

Observation 3

The manufacturer prefers that the retailer starts the markdown sale at $t_m = 0$.

Observation 4

The overall profit of the system also depends on the retailer's start time of the markdown sale. In Case 1, the system profit reached the maximum value at $t_m = 0$, but the retailer benefited from starting the markdown sale at another time.

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- Supply chain contract
 - Change the profit structure to reach the *Pareto* optimum.

- Stackelberg game
 - Leader (manufacturer)
 - ✓ determines the contract parameters
 - Follower (retailer)
 - ✓ determines a start time of the markdown sale and an order quantity

- Revenue-sharing contract
 - Transfer payment from a retailer to a manufacturer
 - ✓ certain fraction of the retailer's revenue ℓ
 - ✓ wholesale price w

■ Revenue-sharing contract

$$\Pi_r(q, t_m) = R_1(q, t_m) + R_2(q, t_m) - (w + c_r) \cdot q - (1 - \ell) \cdot (R_1(q, t_m) + R_2(q, t_m))$$

$$\Pi_m(q, t_m) = (1 - \ell) \cdot (R_1(q, t_m) + R_2(q, t_m)) + wq - c_m q$$

■ Profit of a system

$$\Pi_s(q, t_m) = R_1(q, t_m) + R_2(q, t_m) - (c_r + c_m) \cdot q$$

when $\ell = \frac{w + c_r}{c_r + c_m}$, $w = -c_r + (c_r + c_m) \cdot \ell$ holds true,

\Rightarrow Supply chain is *coordinated*.

- $q_r^* = q_m^* = q_s^*$
- Profit of the system \uparrow

■ Numerical examples for comparison between decentralized and centralized systems

	Case 1	Case 2	Case 3	Case 4
Start time of markdown sale	$t = 17.44$	$t = 18.12$	$t = 0.00$	$t = 20.00$
Order quantity	1,179	1,695	2,966	3,836
Profit of the retailer	59,496	91,034	104,927	290,781
Profit of the manufacturer	12,969	18,654	8,898	46,032
Profit of the system	72,465	109,688	113,825	336,813

	Case 1 ($\ell = 0.75$)	Case 2 ($\ell = 0.83$)	Case 3 ($\ell = 0.92$)	Case 4 ($\ell = 0.86$)
Start time of markdown sale	$t = 0.00$	$t = 17.62$	$t = 0.00$	$t = 20.00$
Order quantity	3,269	1,809	2,982	4,115
Profit of the retailer	67,260	91,523	104,946	290,998
Profit of the manufacturer	22,420	18,746	8,903	47,372
Profit of the system	89,680	110,269	113,849	338,370

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■ Managerial insights

1. It is difficult to match each preference of a start time of the markdown sale in the decentralized system.
2. The preferred start time of the markdown sale from the retailer and manufacturer can be coincided, but the supply chain is not coordinated.
3. The supply chain coordination is achieved when the optimal combination (t_m^{**}, q^{**}) is realized.

감사합니다.